Time Limit: 60 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No Calculators.
Useful Formula: $\cos(\frac{\pi}{5}) = \frac{1+\sqrt{5}}{4}$

1. Given the following two polynomials:
$$x^2 + ax + 1$$
$$x^2 + x + a$$
Find the sum of all real numbers a such that these two polynomials have a common root.
Answer: 1

2. Let x and y be real numbers whose sum is $\frac{11}{12}$ and whose product is $\frac{1}{8}$. Compute the following sum:
$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} x^i y^j$$
Answer: $\frac{24}{5}$

3. Suppose x and y are integers such that $xy + 6x + 5y = 49$. Find the sum of all possible values of x.
Answer: -20

4. Let $f(x) = x^7 - x^5 + x^2 - 1$. Suppose r and s be two non-real roots of $f(x)$. The maximum value of $|r - s|^2$ can be expressed in the form $\frac{a+\sqrt{b}}{c}$ where b is an integer not divisible by any perfect square other than 1. Find $a+b+c$. (Note that the absolute value of a complex number is given by the square root of the sum of the complex number's real part squared and imaginary part squared)
Answer: 12

5. Given that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ find the value of $\sum_{m=1,3,5...}^{\infty} \frac{1}{m^2}$, where m ranges over all odd positive integers.
Answer: $\frac{\pi^2}{8}$

6. This question was omitted from grading due to an error in the question statement.

7. Suppose a_1, a_2, a_3, a_4, a_5 are the roots of the polynomial $x^5 + 7x^4 - 290x^3 - 2030x^2 + 20449x - 143143$, what is the value of $a_1^3 + a_2^3 + a_3^3 + a_4^3 + a_5^3$?
Answer:-343

8. Compute the following sum:
$$\sum_{n=1}^{\infty} \frac{n^2}{2^n}$$
Answer: 6

9. How many functions $f : \mathbb{C} \to \mathbb{C}$ (functions defined from the complex numbers to the complex numbers) satisfy the following properties:
   (i) For all $x, y \in \mathbb{C}$, $|x - y| = |f(x) - f(y)|$
   (ii) If $x^{2017} = 1$ then $f(x)^{2017} = 1$.

   Answer: 4034

10. Let S_n be a collection of n, not necessarily distinct, real numbers in the interval $(1, 12)$. What is the smallest integer n that guarantees there are three values in S_n that could be the side lengths of an acute triangle?
Answer: 12